Exam Introduction to Mathematics Part 3:

Mathematical Modelling - Dimensional Analysis

October 31, 2014: 9.00-10.00.

This exam has 2 problems. Each problem is worth 5 points; more details can be found below. Write on each page your name and student number, and on the first page your seminar group. The use of annotations, books and calculators is not permitted in this examination. All answers must be supported by work. Success.

1. A flat body is sliding on water at a constant speed V. We wish to find the force F resisting such motion. Note that this force is parallel to the water surface. Here it is assumed that the body does not break the water's surface; it merely floats. It is further assumed that the force F depends on the velocity V, the wetted area A of the body, the density ρ of water, and the kinematic viscosity ν of water. The variables can be broken down into the fundamental dimensions lenght L, time T and mass M.

   (a) (*1 point*) The dimension of the kinematic viscosity is L^2T^{-1}. Give the dimension of the other variables.

   (b) (*4 points*) In mathematical terms, the modeling assumption is $F = f(V, A, \rho, \nu)$. Use dimensional analysis to show that this model can be reduced to

   $$\frac{F}{\rho A V^2} = G\left(\frac{\nu}{V\sqrt{A}}\right)$$

   where G is an arbitrary function (that is to be determined in a later stage).

2. The velocity $v(t)$ of waves on a deep ocean satisfies the equation

   $$\frac{dv}{dt} + kv^2 = \ell v$$

   for time $t > 0$, where $v(0) = V$.

   (a) (*1 point*) What are the dimensions of the constants k, ℓ, and V?

   (b) (*2 points*) The above initial value problem is rewritten by introducing a change of variables: $t = t_c s$, $v = v_c u$, where t_c and v_c are constants that have the dimensions of time and velocity, respectively, and they represents a characteristic value of t and v, respectively. Find the dimensionless groups appearing in the transformed problem.

   (c) (*2 points*) Assuming a weak nonlinearity (i.e., the term kv^2 is small in comparison to the other terms in the above differential equation), use the Rules of Thumb given by Holmes to decide on what to take for t_c and v_c.